International Journal of Theoretical & Applied Sciences, **6**(2): 47-56(2014)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Description of Superdeformed Bands of Odd-A and Odd-Odd Mercury and Thallium Nuclei

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(Received 06 July, 2014, Accepted 29 July, 2014)

ABSTRACT: The superdeformed (SD) bands of odd-A and odd-odd Hg–Tl nuclei in the mass region A≈190 have been described well by the Bohr-Mottelson two-term formula as a model. The concepts of the transition energy ratios and E-Gamma Over Spin (EGOS) are used to assign the theoretical level spins. Fitting search program has been employed to extract the model parameters. The best fitted model parameters and the determined spins are used to calculate the E2-transition energies, the rotational frequencies, the kinematic and dynamic moments of inertia. The calculated results agree excellently with the experimental data. The appearance of ΔI = 2 staggering effects in the transition energies of ^{194}Tl (SD1, SD3, SD5) are investigated and examined by using the finite difference approximations to the fourth order derivative of the gamma ray transition energies.
The transition energies against spins for the signature partner pair ^{191}Hg (SD2, SD3) is represented after subtraction of rigid rotor reference. The ΔI = 1 staggering in the odd- A signature partner pairs ^{191}Hg (SD2, SD3), ^{193}Tl (SD1, SD2) is investigated by extracting $\Delta^2E_\gamma(I)$ which represent the difference between the average transitions I+2 → I → I-2 energies in one band and the transition I+1→I-1 energies in its signature partner.

Keywords: Nuclear Structure; Superdeformed Nuclei; Staggering

I. INTRODUCTION

More than 330 superdeformed (SD) bands were studied in nuclei in various regions A ≈ 30, 60,80, 130, 150, 160, 190, 230 and 240 0f nuclear chart [1, 2]. They are associated with extremely large quadrupole deformation. Study of these bands is interesting both theoretically and experimentally. The SD bands observed in various mass regions have their own characteristic features. The difference between the SD bands in various mass region are apparent through the behavior of dynamical moment of inertia $J^{(2)}$. Most SD bands in the A ≈ 190 region exhibit the same increasing trends in $J^{(2)}$ with increasing rotational frequency, while the $J^{(2)}$ pattern near A ≈ 150 show different variations which were shown to be a characteristic of intruder orbital configuration.

Several unexpected features were observed in SD bands, such as the spin, parity and excitation energies of the levels were not measured till now and the spin assignment represent a difficult and unsolved problem. Several fitting procedures for spin assignment were proposed [3-5]. In our previous publications [6-13], we have developed some simple collective models to determine the spins in mass 190, 150, and 60 regions.

One of the most striking and unexpected feature, the phenomenon of identical bands (IB'S). It was first discovered in the nucleus ^{151}Tb (β_2 ≈ 0.6) [14],for which the gamma ray transition energies of the first excited SD band were found to be within 2 KeV of the transition in the yrast SD band of ^{152}Dy. Since the E2 transition energies with ΔI = 2 is very nearly twice the rotational frequency, this means that the rotational frequencies of the two bands are very similar and also implies that the dynamical moments of inertia are almost equal [15]. Several groups tried to understand this phenomenon in framework of phenomenological and semi phenomenological methods [16-18].

It was found that some SD bands, show a slight ΔI = 2 staggering in the gamma-transition energies [19-23] (also called ΔI = 4 bifurcation), i.e. the band energy of spin sequences I = I_0 +4n (n = 0, 1, 2, 3,……..) is somewhat displaced relative to the spin sequence I` = I_0 + 4n +2. The magnitude of the displacement is in the range of some hundred eV to a few KeV. It was suggested that the staggering effects are due to the presence of a hexadecapole perturbation of the prolate SD shape.

Many ΔI = 1 staggering were observed in normal deformed (ND) nuclei for different bands, like odd-even staggering in the gamma vibrational band at low spin [24], the beat odd-even ΔI = 1 staggering patterns observed in the octupole bands [25,26] and the ΔI = 1 odd-even staggering structure of alternating parity bands in even-even nuclei [27,28].

There is another kind of staggering happens in SD odd-A nuclei, the ΔI = 1 staggering in signature partner pairs [29-31]. Most of these signature partners show large amplitude signature splitting and the band head moments of inertia of each pair are almost identical.

The purpose of this paper is two fold. The first is to determine the band head spins of some selected SD bands in A≈ 190 mass region. The second objective is to study the properties of moments of inertia and the ΔI = 2, ΔI = 1 staggering effects is our selected SD bands.

The paper is organized as follows: following this introduction, we describe the formalism of our approach in section 2. In section 3 we suggest a method to assign the band head spins of the SD bands. Section 4 is devoted to explore the Δ I = 2 staggering in A ≈ 190 mass region. Section 5, concerns the origin of Δ I = 1 staggering in signature partner pairs in odd-A SD bands. In section 6, we present the numerical calculations and the obtained results for seven SD bands in Mercury and Thallium nuclei, discussion are also included. Finally, conclusion and remarks are given in section 7.

II. OUTLINE OF THE MODEL

One of the earliest attempts involved the addition of second term to the simple rotational formula of the rigid rotor, and one can express the rotational energies E (I) of state of spin I of an axially symmetric deformed nucleus under the adiabatic approximation by the Bohr-Mottelson two-term formula [32]:

$$E(I) = A\,[I(I+1)] + B[I(I+1)]^2 \qquad (1)$$

Where A is the common inertial parameter $A = \frac{\hbar^2}{2\mathcal{J}}$, with $\mathcal{J}$ is the moment of inertia, B is commonly negative and almost 10^3 times less than the value of A.

The gamma –ray transition energies within a band has the form:

$$E_\gamma(I) = E(I) - E(I-2)$$
$$= A\,[2(2I-1)] + B\,[4(2I-1)(I^2-I+1)] \qquad (2)$$

The ratio of $E_\gamma(I)$ over 2(2I-1), (E- Gamma Over Spin or EGOS) is given by:

$$EGOS = \frac{E_\gamma(I)}{2(2I-1)} = A + B\,[2(I^2-I+1)] \qquad (3)$$

An EGOS plot is thus simply $E_\gamma/2(2I-1)$ plotted against $2\,(I^2 - I + 1)$ which give a straight line of slope B and intersect A.

In the framework of the collective rotational models, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia for the expression E(I) equation (1) reads:

$$J^{(1)}(I) = \hbar^2\left[\frac{1}{\sqrt{I(I+1)}}\frac{dE(I)}{d\sqrt{I(I+1)}}\right]^{-1}$$
$$= \frac{\hbar^2}{2}[A + 2B\,I(I+1)]^{-1} \qquad (4)$$

$$J^{(2)}(I) = \hbar^2\left[\frac{d^2E(I)}{d\left(\sqrt{I(I+1)}\right)^2}\right]^{-1}$$
$$= \frac{\hbar^2}{2}[A + 6B\,I(I+1)]^{-1} \qquad (5)$$

Also, the rotational frequency ħω is given by:

$$\hbar\omega(I) = \frac{dE(I)}{d\sqrt{I(I+1)}}$$
$$= 2A\sqrt{I(I+1)}\left[1 + \frac{2B}{A}I(I+1)\right] \qquad (6)$$

Experimentally, for the SD bands, only gamma – ray transition energies are available and are commonly translated into values of ħω and $J^{(2)}$ as:

$$\hbar\omega\,(I) = \frac{[E_\gamma(I) + E_\gamma(I+2)]}{4} \qquad (7)$$

$$J^{(2)}(I) = \frac{4}{E_\gamma\,(I+2) - E_\gamma\,(I)} \qquad (8)$$

Also the kinematic moment of inertia $J^{(1)}$ can be extracted from the gamma – ray transition energies as:

$$J^{(1)}(I) = \frac{2I-1}{E_\gamma\,(I)} \qquad (9)$$

It is seen that, while $J^{(1)}$ depends on spin I, $J^{(2)}$ does not.

III. SPIN ASSIGNMENT FOR SD BANDS

As a first –hand approximation for band head spin assignment, we use the ratio between the two transition energies E_γ ($I_0 + 4 \rightarrow I_0$) and E_γ ($I_0 + 2 \rightarrow I_0$)

$$R = \frac{E_\gamma\ (I_0+4 \rightarrow I_0\ )}{E_\gamma (I_0+2 \rightarrow I_0)} (10)$$

For rigid rotor

$$E(I) = A\,I\,(I+1) \quad (11)$$

The ratio R becomes

$$R = \frac{4\,I_0 + 10}{2I_0 + 3} \quad (12)$$

From which the band head I_0 can be determined

$$I_0 = \frac{10-3R}{2R-4} (13)$$

For our two –term formula, equation (2), the ratio R becomes

$$R = \frac{(4I_0+10)[1+\lambda(I_0^2+5I_0+10)]}{(2I_0+3)[1+\lambda(I_0^2+3I_0+3)]} (14)$$

with $\lambda = 2B / A$

IV. THE Δ I = 2 STAGGERING PHENOMENON

To explore the Δ I = 2 staggering, for each band the deviation of the transition energies from a smooth reference ΔE_γ was determined by calculating the finite difference approximation of the fourth order derivative of the transition energies at given spin $d^4E_\gamma(I) / dI^4$.This smooth reference is given by [10]:

$$\Delta^4 E_\gamma (I) = \frac{1}{16}\left[E_\gamma(I+4) - 4E_\gamma(I+2) + 6E_\gamma(I) - 4E_\gamma(I-2) + E_\gamma(I-4)\right] \qquad (15)$$

This formula includes five consecutive transitions energies $E_\gamma$and is denoted by five –point formula.

V. Δ I = 1 STAGGERING IN SIGNATURE PARTNER PAIRS OF ODD-A SD BANDS

To investigate the Δ I =1 staggering in signature partner pairs of odd-A SD bands ,one must extract the differences between the average transitions E_γ ($I+2 \rightarrow I$) and E_γ($I \rightarrow I-2$) energies in one band and the transition E_γ ($I+1 \rightarrow I-1$) energies in the signature partner:

$$\Delta^2 E_\gamma(I) = \frac{1}{2}\left[E_\gamma(I+2 \rightarrow I) + E_\gamma(I \rightarrow I-2) - 2E_\gamma(I+1 \rightarrow I-1)\right] \qquad (16)$$

VI. NUMERICAL CALCULATIONS AND DISCUSSION

To predict the band head spin for each SD band, we use equation (13) as a first – hand approximation, the ratio between two observed intraband gamma transition energies assuming that the band is purely rotational.

Then, as a second-hand approximation, we investigate the variation of the transition energies in framework of Bohr –Mottelson two-term formula equation (14). The procedure is repeated for several trial values of I_0 of the spin of the lowest observed level and the model parameters A and B are fitted to reproduce the gamma transition energies. The values of I_0, A and B which leads to the minimum of root-mean-square (rms)deviation are chosen.

$$\chi = \left[\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E_\gamma^{exp}(I_i) - E_\gamma^{cal}(I_i)}{E_\gamma^{exp}(I_i)}\right|^2\right]^{1/2}$$

Table 1 summarize the values of band head spin I_0, the lowest transition energy E_γ ($I_0 + 2 \rightarrow I_0$), the model parameters A and B obtained by best fitting procedure also the band head moment of inertia J_0 and the transition energy ratio R are given for our seven selected SD bands namely: ^{194}Tl (SD1,SD3, SD5),^{191}Hg (SD2, SD3) and ^{193}Tl (SD1, SD2).

Using the adopted I_0, A and B, the results for the transition energies E_γ(I), the rotational frequency ħω, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are given in Table 2.

Fig. 1 illustrate the behavior of $J^{(1)}$ (open circles) and $J^{(2)}$ (closed circles) as a function of ħω. Investigating the table and the figure, it is seen that the agreement between calculated and the experimental data (closed circles with error bars) is excellent. $J^{(1)}$ and $J^{(2)}$ shows a smooth and similar increase with ħω, which can be understood by three effects: the gradual angular momentum alignment of a pair of $j_{15/2}$ neutrons, the angular momentum alignment of a pair of $i_{13/2}$ protons at a somewhat higher frequency and from the gradual disappearance of pairing correlations.

Another result of the present work is the observation of a ΔI = 2 staggering effects in the gamma ray transition energies E_γ (I) of ^{194}Tl (SD1, SD3, SD5). The staggering parameter Δ^4E_γ(I) has been calculated by using the finite difference approximation outlined in section4.The staggering parameters Δ^4E_γ(I) for each band are shown in Table 3 and Fig. 2 as a function of nuclear spin I and rotational frequency ħω. A significant anomalous staggering has been observed. Now, we would like to focus on ΔI =1 staggering phenomenon in odd- A SD signature partner pairs ^{191}Hg (SD2,SD3) and ^{193}Tl (SD1, S2). The energy shift Δ^2E_γ (I) values have been extracted and listed in Table 4, and plotted versus spin I in Fig. 3.

Table 1: The spin proposition I_0, the band head moment of inertia J_0, the adopted best model parameters A, B (obtained from the fit) and the transition energy ratio R for our selected SD bands.

SD band	I_0 (ħ)	J_0 ($\hbar^2 MeV^{-1}$)	A (KeV)	B (KeV)	R
Odd-Odd nuclei					
^{194}Tl (SD1)	12	99.732	5.0134	-1.3132×10^{-4}	2.144
(SD3)	10	95.270	5.2482	-1.9806×10^{-4}	2.169
(SD5)	8	101.514	4.9254	-1.2103×10^{-4}	2.208
Odd-A nuclei					
^{191}Hg(SD2)	10.5	93.986	5.3199	-2.1063×10^{-4}	2.161
(SD3)	11.5	94.146	5.3109	-2.3083×10^{-4}	2.148
^{193}Tl (SD1)	8.5	95.715	5.2238	-2.1146×10^{-4}	2.195
(SD2)	9.5	95.743	5.2223	-1.8055×10^{-4}	2.177

Table 2: The calculated transition energies Eγ, the rotational frequency ħω, dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia and comparison with experimental data [1,2].

Assigned I(ħ)	Eγ (cal) (KeV)	ħω (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)	$J^{(2)}_{exp}$ ($\hbar^2 MeV^{-1}$)	E_γ(exp) (KeV)
^{194}Tl (SD1)						
14	268.128	0.1437	103.151	100.698	102.564	268.0
16	306.906	0.1630	104.201	101.008	104.986	307.0
18	345.293	0.1821	105.410	101.363	102.301	345.1
20	383.240	0.2009	106.797	101.708	108.695	384.2
22	420.694	0.2195	108.362	102.212	111.111	421.0
24	457.607	0.2378	110.135	102.708	105.540	457.0
26	493.926	0.2558	112.117	103.254	111.111	494.9
28	529.603	0.2735	114.337	103.851	110.803	530.9
30	564.587	0.2908	116.822	104.501	116.959	567.0
32	598.827	0.3077	119.595	105.205	118.694	601.2
34	632.273	0.3242	122.699	105.966	114.613	634.9
36	664.873	0.3403	126.159	106.787	118.343	669.8
38	696.579			107.669		703.6
^{194}Tl (SD3)						
12	238.993	0.1296	98.737	96.237	101.265	240.5
14	279.488	0.1495	100.047	96.605	103.092	280.0
16	319.469	0.1695	101.543	97.036	101.781	318.8
18	358.861	0.1891	103.289	97.530	102.301	358.1
20	397.587	0.2082	105.304	98.091	104.986	397.2
22	435.572	0.2270	107.622	98.720	106.100	435.3
24	472.739	0.2454	110.271	99.420	105.540	473.0
26	509.013	0.2633	113.301	100.193	112.044	510.9
28	544.317	0.2807	116.761	101.044	112.359	546.6
30	578.575	0.2970	120.761	101.974	113.636	582.2

32	611.711	0.3138	125.242	102.989	115.606	617.4
34	643.649	0.3294	130.441	104.094	119.402	652.0
36	674.314	0.3444	136.453	105.292	125.000	685.5
38	703.627			106.590		717.5
^{194}Tl (SD5)						
10	186.328	0.1028	103.201	101.970	104.166	187.9
12	225.087	0.1221	103.917	102.182	106.100	226.3
14	263.579	0.1413	104.772	102.436	105.263	264.0
16	301.757	0.1603	105.766	102.731	107.526	302.0
18	339.576	0.1791	106.920	103.069	106.951	339.2
20	376.987	0.1977	108.228	103.451	107.816	376.6
22	413.946	0.2160	109.715	103.878	110.192	413.7
24	450.404	0.2341	111.380	104.350	110.803	450.0
26	486.317	0.2519	113.250	104.869	112.044	486.1
28	521.637	0.2694	115.336	105.437	109.289	521.8
30	556.318	0.2866	117.660	106.054	113.314	558.4
32	590.314	0.3034	120.253	106.722	117.647	593.7
34	623.577			107.444		627.7
^{191}Hg(SD2)						
12.5	252.426	0.1364	97.933	95.076	99.255	252.4
14.5	293.272	0.1567	99.307	95.474	99.009	292.7
16.5	333.551	0.1766	100.931	95.937	100.882	333.1
18.5	373.182	0.1963	102.817	96.467	102.432	372.75
20.5	412.086	0.2155	105.000	97.067	103.896	411.08
22.5	450.181	0.2343	107.512	97.738	105.820	450.3
24.5	487.386	0.2527	110.393	98.484	107.816	488.1
26.5	523.620	0.2706	113.691	99.308	109.890	525.2
28.5	558.803	0.2879	117.470	100.214	112.359	561.6
30.5	592.854	0.3046	121.813	101.205	114.449	597.2
32.5	625.691	0.3207	126.811	102.286	117.474	632.15
34.5	657.234	0.3361	132.590	103.463	118.694	666.2
36.5	687.402	0.3308	139.309	104.742	121.951	699.9
38.5	716.115	0.3648	147.194	106.128	123.076	732.7
40.5	743.290	0.3780	156.506	107.629	127.795	765.2
42.5	768.848			109.254		796.3
^{191}Hg(SD3)						
13.5	272.091	0.1461	99.233	95.556	97.323	272.0
15.5	312.400	0.1661	100.895	96.030	101.394	313.1
17.5	352.045	0.1857	102.854	96.578	102.695	352.55
19.5	390.935	0.2049	105.127	97.202	104.712	391.5
21.5	428.984	0.2237	107.764	97.905	106.951	429.7
23.5	466.102	0.2420	110.806	98.690	108.695	467.1
25.5	502.201	0.2598	114.315	99.561	111.576	503.9
27.5	537.192	0.2770	118.364	100.522	113.475	539.75
29.5	570.986	0.2936	123.042	101.578	116.110	575.0
31.5	603.459	0.3059	128.472	102.734	120.300	609.45
33.5	634.630	0.3247	134.807	103.997	119.760	642.7
35.5	664.302	0.3391	142.237	105.373	123.456	676.1
37.5	692.424	0.3528	151.051	106.870	126.984	708.5
39.5	718.905	0.3656	161.596	108.498	127.795	740.0
41.5	743.658	0.3775	174.398	110.265	136.986	771.3
43.5	766.594					800.5

Assigned I(ħ)	Eγ (cal) (KeV)	ħω (MeV)	$J^{(2)}$ ($\hbar^2 MeV^{-1}$)	$J^{(1)}$ ($\hbar^2 MeV^{-1}$)	$J^{(2)}_{exp}$ ($\hbar^2 MeV^{-1}$)	E_γ(exp) (KeV)
^{194}Tl (SD1)						
14	268.128	0.1437	103.151	100.698	102.564	268.0
16	306.906	0.1630	104.201	101.008	104.986	307.0
18	345.293	0.1821	105.410	101.363	102.301	345.1
20	383.240	0.2009	106.797	101.708	108.695	384.2
22	420.694	0.2195	108.362	102.212	111.111	421.0
24	457.607	0.2378	110.135	102.708	105.540	457.0
26	493.926	0.2558	112.117	103.254	111.111	494.9
28	529.603	0.2735	114.337	103.851	110.803	530.9
30	564.587	0.2908	116.822	104.501	116.959	567.0
32	598.827	0.3077	119.595	105.205	118.694	601.2
34	632.273	0.3242	122.699	105.966	114.613	634.9
36	664.873	0.3403	126.159	106.787	118.343	669.8
38	696.579			107.669		703.6
^{194}Tl (SD3)						
12	238.993	0.1296	98.737	96.237	101.265	240.5
14	279.488	0.1495	100.047	96.605	103.092	280.0
16	319.469	0.1695	101.543	97.036	101.781	318.8
18	358.861	0.1891	103.289	97.530	102.301	358.1
20	397.587	0.2082	105.304	98.091	104.986	397.2
22	435.572	0.2270	107.622	98.720	106.100	435.3
24	472.739	0.2454	110.271	99.420	105.540	473.0
26	509.013	0.2633	113.301	100.193	112.044	510.9
28	544.317	0.2807	116.761	101.044	112.359	546.6
30	578.575	0.2970	120.761	101.974	113.636	582.2

Table 3. The calculated ΔI = 2 staggering parameter $\Delta^4 E\gamma(I)$ of the SD bands 1,3,5 in ^{194}Tl .

^{194}Tl(SD1)			^{194}Tl(SD3)			^{194}Tl(SD5)		
I (ħ)	ħω (MeV)	Δ^4Eγ(I) (KeV)	I (ħ)	ħω (MeV)	Δ^4Eγ(I) (KeV)	I (ħ)	ħω (MeV)	Δ^4Eγ(I) (KeV)
18	0.1821	-5.2	16	0.1695	-1.9	14	0.1413	-2.1
20	0.2009	4.8	18	0.1891	-4.2	16	0.1603	2.1
22	0.2195	1.2	20	0.2082	1.8	18	0.1791	-1.5
24	0.2378	-6.5	22	0.2270	-0.6	20	0.1977	0
26	0.2558	5.8	24	0.2454	-2.6	22	0.2160	1.1
28	0.2735	-4.0	26	0.2633	4.4	24	0.2341	-0.8
30	0.2908	3.4	28	0.2807	-2.4	26	0.2519	1.5
32	0.3077	0.3	30	0.2970	0.1	28	0.2694	-3.5
34	0.3242	-4.0	32	0.3138	-0.3	30	0.2866	2.2

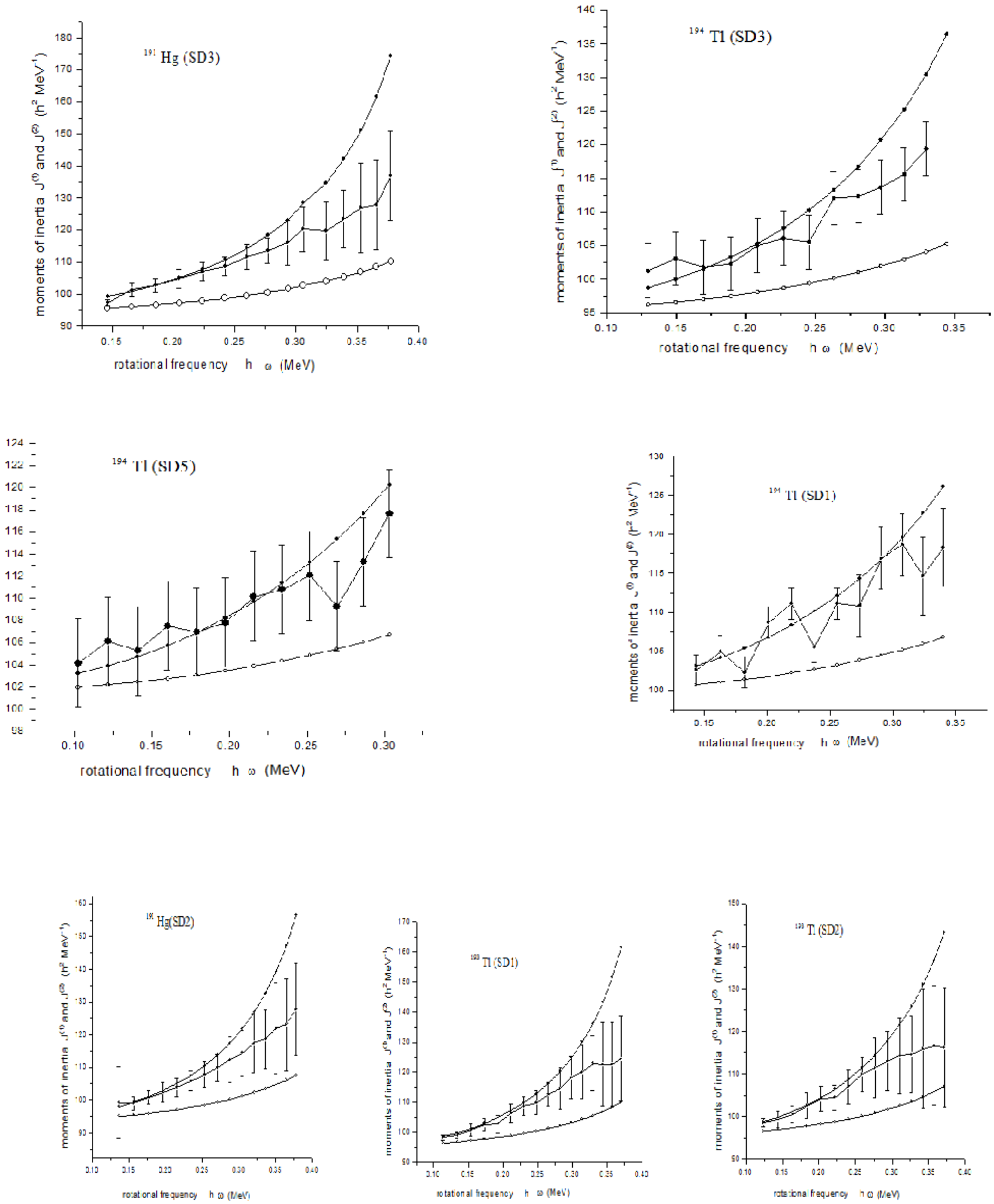

Fig.1. The calculated results of kinematic $J^{(1)}$ (open circles) and dynamic $J^{(2)}$ (closed circles) moments of inertia as a function of the rotational frequency ħω for the SD bands ^{194}Tl (SD1,SD3,SD5) , ^{191}Hg(SD2,SD3) and ^{193}Tl (SD1,SD2) and comparison with experimental data (closed circles with error bars).

Table 4: The calculated ΔI = 1 staggering parameter $\Delta^2E\gamma(I)$ for the signature partner pairs ^{191}Hg (SD2,SD3), ^{193}Tl(SD1, SD2).

^{191}Hg (SD2 ,SD3)		^{193}Tl(SD1,SD2)	
I(ħ)	$\Delta^2E\gamma(I)$ (KeV)	I(ħ)	$\Delta^2E\gamma(I)$ (KeV)
13.5	1.518	11.5	-0.642
14.5	-2.053	12.5	0.612
15.5	2.023	13.5	-1.068
16.5	-2.657	14.5	1.072
17.5	2.643	15.5	-1.646
18.5	-3.381	16.5	1.701
19.5	3.398	17.5	-2.405
20.5	-4.253	18.5	2.520
21.5	4.299	19.5	-3.365
22.5	-5.276	20.5	3.550
23.5	5.363	21.5	-4.548
24.5	-6.469	22.5	4.819
25.5	6.604	23.5	-5.983
26.5	-7.847	24.5	6.350
27.5	8.039	25.5	-7.691
28.5	-9.428	26.5	8.165
29.5	9.685	27.5	-9.695
30.5	-11.227	28.5	10.288
31.5	11.555	29.5	-12.018
32.5	-13.257	30.5	12.744
33.5	13.665	31.5	-14.688
34.5	-15.536	32.5	15.557
35.5	16.032	33.5	-17.725
36.5	-18.078	34.5	18.749
37.5	18.669	35.5	-21.153
38.5	-20.901	36.5	22.344
39.5	21.595	37.5	-24.997
40.5	-24.017	38.5	26.367
41.5	24.822	39.5	-29.280
42.5	-27.444	40.5	30.841
		41.5	-34.026

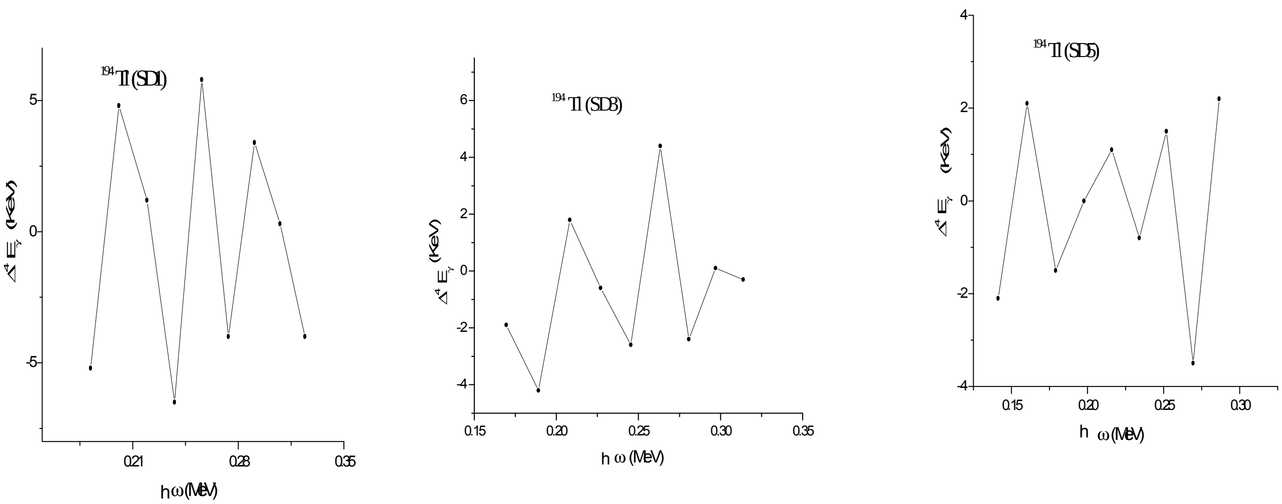

Fig. 2. The calculated ΔI = 2 staggering parameter $\Delta^4E\gamma$ (I) as a function of the rotational frequency ħω for the SD bands ^{194}Tl (SD1,SD3,SD5).

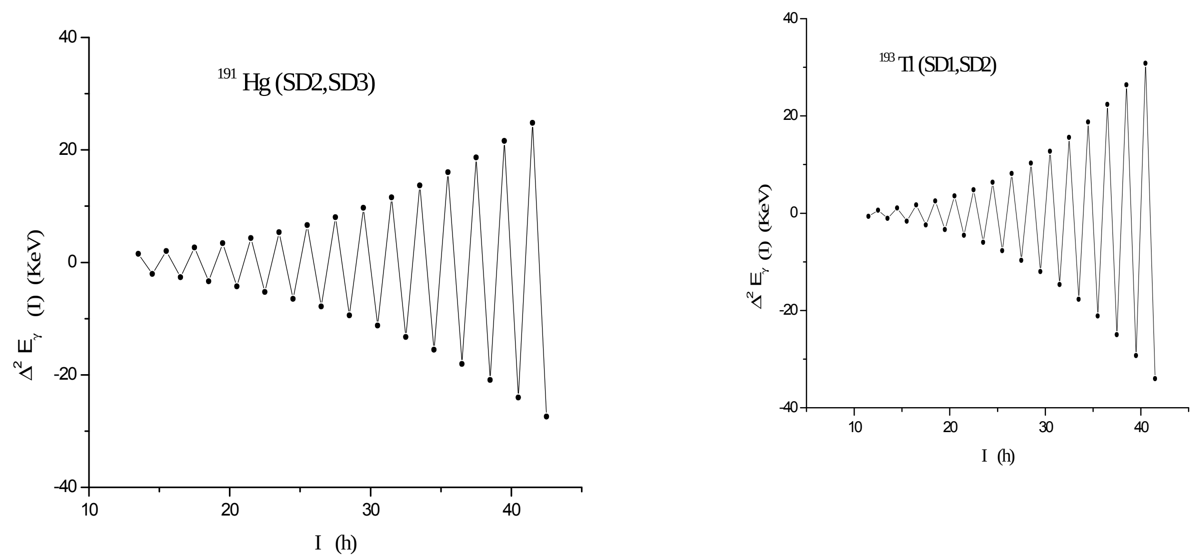

Fig. 3. The calculated ΔI = 1 staggering parameter $\Delta^2E\gamma$(I) as a function of spin I for the signature partner pairs ^{191}Hg (SD2 ,SD3), ^{193}Tl(SD1,SD2).

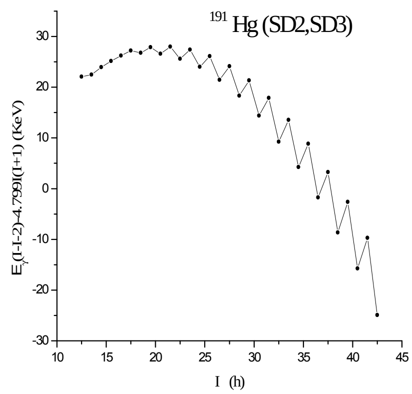

Fig. 4. The ΔI = 1 staggering in the calculated transition energies minus rigid rotor reference as a function of spin I for the signature partner pair ^{191}Hg (SD2, SD3).

The signature partner pair ^{191}Hg (SD2, SD3) has been interpreted as signature partners built on the 312^{+}[642] orbital. For the signature partner ^{193}Tl (SD1, SD2), the saturation of the dynamical moment of inertia $J^{(2)}$ at the rotational frequency ħω> 0.3 MeV is observed for the two bands, reflecting the combined effects of the proton pairing blocking and complete $j_{15/2}$ neutron alignment. It is interesting to note that the band head moments of inertia of each signature partner pair are almost similar (Table 1).

Another ΔI =1 Staggering happen in the transition energies Eγ (I) after subtracting a rigid rotor reference, when plotted versus spin for the signature partner pair ^{191}Hg (SD2, SD3), the result is shown in Fig. 4. It shows that E_{γ}(I) of band 2 shift distinctly from the midpoint of band 3, a zigzag pattern emerges.

VII. CONCLUSION

We showed in this paper that the SD nuclear states of odd- A and odd-odd Hg –Tl nuclei can be described with Bohr-Mottelson two –terms formula, which is quite successful in explaining the normally deformed (ND) nuclei. For each SD band the band head spin is determined and the two parameters of the model are fitted to reproduce the observed gamma ray transition energies. The E2 transition energies, the dynamic and kinematic moments of inertia are calculated. The calculated results agree with experimental data vey well. We found a ΔI = 2 staggering in the three SD bands in odd-odd nucleus ^{194}Tl by performing staggering parameter analysis. TheΔI = 1 staggering in the two signature partner pairs^{191}Hg (SD2, SD3) and ^{193}Tl (SD1, SD2) are investigated by extracting the difference between the average I+2 → I → I-2 transition energies in one band and the I+1 → I-1 transition energies in its signature partner.

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